Electron Density and Radial Probability Plots

A Chem 101A Tutorial
The “master equation” that governs the behavior of matter at the atomic scale is the **Schrödinger equation**: 

\[ \hat{H}\psi = E\psi \]
The Schrödinger equation contains two “variables”, $E$ and $\psi$. $E$ is the **internal energy** of the atom, which is the sum of the potential energies and kinetic energies of all of the electrons.

$\psi$ is a **wave function**.

The Schrödinger equation has infinite solutions, each of which is an energy and a corresponding wave function.

This is similar to an algebraic equation in two unknowns. For instance, the equation $x + y = 5$ has infinite solutions, with each solution containing an $x$ value and the corresponding $y$ value:

$$
\begin{align*}
  x &= 0, y = 5 \\
  x &= 3, y = 2 \\
  x &= 17, y = -12 \\
  x &= -50.23356, y = 55.23356 \\
  x &= 17, y = -12 \\
  x &= \sqrt[3]{293}, y = 5 - \sqrt[3]{293} \\
  \text{etc.}
\end{align*}
$$
The symbol $\hat{H}$ in the Schrödinger equation stands for a set of mathematical instructions, the details of which are beyond the level of Chem 101A. (You’ll need to take differential equations before these instructions make much sense to you.)

However, the basic idea of Schrödinger’s equation is this: “Look for a function ($\psi$) that you can do a bunch of weird stuff ($\hat{H}$) to and end up with the original function, just multiplied by a number ($E$).”

$$\hat{H}\psi = E\psi$$
What do solutions to the Schrödinger equation look like?

For a hydrogen atom, one solution is:

\[ E = -2.180 \times 10^{-18} \text{ J} \]
\[ \psi = (0.0014363 \text{ pm}^{-3/2}) e^{-r/(52.9 \text{ pm})} \]

Another solution is:

\[ E = -1.362 \times 10^{-19} \text{ J} \]
\[ \psi = (2.149 \times 10^{-9} \text{ pm}^{-3/2})(x^2 + y^2)[(634.8 \text{ pm})r^2 - r^3]e^{-r/(211.6 \text{ pm})} \]

In these equations, \( r \) is the distance from the nucleus to the electron (which equals \( \sqrt{x^2 + y^2 + z^2} \) in normal Cartesian coordinates), and \( x \) and \( y \) represent the \( x \) and \( y \) coordinates of the electron’s location, with all distances being in picometers (pm). Don’t worry - you won’t be asked to learn or use these formulas!
The values of $E$ are the allowed energies for the electron, but what do these wave functions mean?

Well, $\psi$ itself has no physical meaning at all. It doesn’t correspond to anything we can observe or measure.

However, $\psi^2$ is a meaningful expression; it gives us the electron density at any point $(x, y, z)$ around the nucleus.

...but this still doesn’t help if we don’t know what “electron density” is.

Okay, enough scary-looking math. Time to look at what all of this means...
Electron density

There are two ways to visualize electron density, based on the dual nature of matter.

Remember that all matter (including electrons) behaves as if it is both a particle and a wave. Depending on what we are measuring, sometimes we observe the electron behaving as a particle, sometimes as a wave.
Think of a wave in the ocean. It is spread out over a substantial area, without any obvious boundaries.

Likewise, an electron in an atom behaves as if it is spread out over a large region around the nucleus, with no evident boundaries.

This is the wave nature of an electron.
For a hydrogen atom in its ground state (the 1s orbital), the “electron wave” looks like a smeared-out, spherical cloud of negatively charged matter, with the nucleus at its center.

The cloud has no outer boundary, but 99% of the cloud lies within 220 pm from the nucleus, and 99.999% of the cloud lies within 430 pm from the nucleus.
The electron density is the concentration of matter at a location in this cloud. This concentration is normally expressed as the amount of matter per cubic picometer.

For example, when a hydrogen atom is in its ground state, the electron density at any point that is 10 pm away from the nucleus is 0.000001413 pm$^{-3}$.

This means that if we construct a 1 pm$^3$ box whose center is 10 pm from the nucleus, 0.000001413 of the electron will be in the box (that’s 0.0001413% of the electron).

The other 0.999998587 of the electron (99.9998587% of the electron) is outside the box.

*(Go on to the next page for a picture of this...)*
"The electron density at a distance of 10 pm is 0.00001413 pm⁻³."

Meaning: 0.00001413 of an electron is inside this 1 pm³ box. (The rest of the electron is outside the box.)

Note: the electron cloud is spherically symmetric, so any 1 pm³ box that is 10 pm away from the nucleus will hold 0.00001413 of the electron.
The electron density is analogous to the percentage of an ocean wave at a particular distance from the shore.

This red box is one foot wide and contains 0.015 of the wave (1.5% of the wave), so the “wave density” 100 feet from the shore is 0.015 per foot, or $0.015 \text{ ft}^{-1}$. 
We can calculate the electron density at any point by calculating $\psi^2$ at that point.

For example, for the ground state of hydrogen,

$$\psi = (0.0014363 \text{ pm}^{-3/2})e^{-r/(52.9 \text{ pm})}.$$ 

To find the electron density at a point that is 10 pm from the nucleus, we substitute $r = 10$ pm into the formula above.

$$\psi = (0.0014363 \text{ pm}^{-3/2})e^{-10 \text{ pm}/(52.9 \text{ pm})}.$$ 

$$= (0.0014363 \text{ pm}^{-3/2})e^{-0.189036}$$ 

$$= (0.0014363 \text{ pm}^{-3/2})(0.82776)$$ 

$$0.0011889 \text{ pm}^{-3/2}.$$ 

Then we square this value:

$$\text{Electron density} = (0.0011889 \text{ pm}^{-3/2})^2$$ 

$$= 0.000001413 \text{ pm}^{-3}$$
Another way to visualize electron density is based on the particle nature of the electron.

Imagine that we could build a camera that can take photographs of a hydrogen atom.

We will take a series of photos, each of which shows the positions of the nucleus (a proton) and the electron.
Here are the first few pictures. The electron (the blue dot) appears in a different location in each photo, sometimes close to the nucleus, sometimes farther away.

Now let’s make these pictures transparent and stack them on top of one another…
Here is the result. Now we can see the positions of the electron in all six of our photos at the same time.

Again, sometimes the electron is close to the proton, sometimes farther away. There is no obvious pattern that we can see, though; the electron seems to wander randomly around the proton.

Now, what if we took many, many photos and piled them all on top of one another??
Here is what we might see after we take 100 photos of the atom and stack them atop one another. Many of our photographs show the electron close to the nucleus - the blue dots are clustered together very thickly. As we move away from the nucleus, the dots are more spread out.
Now let us take a 1 cubic pm box and put it over our photos….
When we put the box close to the nucleus, the electron often passes through it; 11 of our photos show the electron in this region.
When we put the box farther from the nucleus, the electron passes through it less frequently. Only 2 of our photos show the electron in this region.
The electron density is the fraction of time that the electron appears in the 1 pm$^3$ box, which is equal to the fraction of photos that show the electron in the box.

Electron density at this distance = 11/100 = 0.11 per cubic picometer (0.11 pm$^{-3}$)

Electron density at this distance = 2/100 = 0.02 per cubic picometer (0.02 pm$^{-3}$)
Going back to our earlier example, we found that the electron density 10 pm from the nucleus is 0.000001413 pm$^{-3}$.

This means that the probability of finding the electron in a 1 pm cube centered 10 pm from the nucleus is 0.000001413.

That is a very small probability, equal to 1 chance in 707,700 (because $1/707,700 = 0.000001413$). In other words, for every 707,700 photos we take of the atom, only one of them will show the electron in this box! (The probability is very small because our box is very small.)
How does the electron density vary within a hydrogen atom?
The ground state of the hydrogen atom is the 1s orbital, so let’s look at that orbital first.

We’ll draw a graph of the electron density versus the distance from the nucleus…

How does the electron density change as we move away from the nucleus?
Here is the graph we obtain. This is the electron density plot for the 1s orbital.

What does this graph mean?? Turn to the next page....
On this graph, the $x$ value gives the distance between the nucleus and the 1 pm$^3$ box where we wish to measure the electron density.
The $y$ value gives the electron density in units of pm$^{-3}$ (i.e. the fraction of the electron we will find in a 1 pm cube).

Note that the electron density values are very small; only a tiny fraction of the electron is found in a 1 pm cube, regardless of where we put the cube.  *(The numbers are written in computer format; 1.0E-06 means 1.0 x 10$^{-6}$)*
The blue curve shows the relationship between distance and electron density. For example, at a distance of 50 pm from the nucleus, the electron density is roughly $3 \times 10^{-7}$ pm$^{-3}$. 
In the 1s orbital, the electron density is highest at the nucleus (where distance = 0 pm). As we move away from the nucleus, the electron density decreases, approaching (but never reaching) zero as the distance becomes very large.
Here is an electron density plot for the 2s orbital, drawn to the same scale as the 1s plot. It looks like the electron density is much lower in the 2s orbital, regardless of the distance from the nucleus. But let us increase the scale…
When we increase the $y$ axis scale by a factor of 10, we see that the electron density does not decrease smoothly as we increase the distance; it drops to zero, then goes up again. Let’s increase the scale still more…
When we increase the $y$ axis scale by another factor of 10, the extra hump becomes obvious. In addition, we can see that the hump continues beyond 400 pm. Let’s extend our $x$ axis scale to 1000 pm....
When we extend the distance out to 1000 pm, we see that the right side of the hump drops smoothly toward an electron density of zero, just as it did for the 1s plot.
This plot shows the 1s and 2s electron density curves plotted at a small scale. The 1s orbital has a much higher electron density close to the nucleus.

This plot shows the 1s and 2s electron density curves plotted at a large scale. The 2s orbital has significant electron density out to around 600 pm, farther than the 1s orbital.

Comparing the 1s and 2s orbitals, we see that the 2s orbital extends farther than the 1s, while the 1s orbital has a much higher electron density close to the nucleus.
Note that at a distance of around 100 pm, the electron density drops to zero in the 2s orbital.

At this point, there is a node, a surface where the electron can never be found.
The node in a 2s orbital is actually a spherical surface whose radius is 105.9 pm. The picture below shows a cross-section through the 2s orbital; the gray shading represents the electron.

The existence of nodes is a wave property of the electron. Part of the “electron wave” is inside the node, and the rest is outside the node.

If we think of the electron as a particle, the meaning of the node is that the electron can be inside the node or outside it, but never on it. (How does it get from the inside to the outside? It just does… Yes, quantum mechanics can be weird.)
All s orbitals are similar to the 1s and 2s orbitals in having their highest electron density at the nucleus and dropping off to zero as the distance becomes very large.

However, higher s orbitals (3s, 4s, 5s, etc.) extend farther from the nucleus (i.e. they are larger).

Also, higher s orbitals have additional nodes. All of these nodes are spheres centered on the nucleus.

We call these spheres **radial nodes**, because they are at fixed radii from the nucleus.
Here is a cross-section of the 3s orbital. This orbital has two radial nodes, the two spheres shown in red.

On the next page, there is an electron density plot of this orbital.
Here is the electron density plot for the 3s orbital. We see the two radial nodes clearly in this graph; one is at 101 pm and the other is at 375 pm from the nucleus.

The electron can be inside the first node (between 0 and 101 pm), between the nodes (between 101 and 375 pm), or outside the second node (beyond 375 pm), but it cannot be on a node: it is never 101 pm or 375 pm from the nucleus.
These graphs compare the three s orbitals we’ve examined so far. The first graph shows that the 1s orbital has the highest electron density near the nucleus (by far), with the 2s being lower and the 3s being still lower.

The second graph shows that the 1s orbital has no nodes and only extends out to around 300 pm. The 2s orbital has one node (at 100 pm) and extends out to around 600 pm. The 3s orbital has two nodes and extends out beyond 1000 pm.

As we increase n, the orbital becomes larger, the number of nodes increases, and the electron density near the nucleus becomes smaller.
Now let us look at orbitals that are not spherical. The picture below shows a cross-section of a 2p orbital. All p orbitals have a node that passes through the nucleus and divides the orbital in half. This type of node is called an **angular node**.

When we draw an electron density plot, we always start at the nucleus. Since there is a node at the nucleus, the electron density is zero there; the plot will begin at the point (0, 0).
Here is the plot for the 2p orbital. It starts at (0, 0), rises to a maximum at about 100 pm from the nucleus, and then drops gradually toward zero as the distance becomes very large.
This graph shows the electron density plots for the 2s and 2p orbitals. The two orbitals are about the same size (they extend out to around 600 pm), but the 2s has its highest electron density near the nucleus, while the 2p has its highest electron density at around 100 pm. Note that each orbital has a node where the other orbital has its highest electron density; the two orbitals complement each other.
Let’s see if we can figure out what the electron density plot looks like for a 3p orbital.

To do so, we must know something about the nodes in this orbital. Here are some general facts about nodes.

1) The total number of nodes in any orbital is \((n - 1)\).

2) The number of angular nodes (planar nodes) in any orbital equals \(\ell\).

3) All remaining nodes are radial nodes (spherical nodes).
For a 3p orbital, \( n = 3 \). (Remember that the name of the orbital always starts with the value of \( n \), the principal quantum number.) Therefore, the 3p orbital has **two nodes**.

For any p orbital, \( \ell = 1 \). Therefore, the 3p orbital has **one angular node**. This node is a plane that divides the orbital in half, just as we saw for a 2p orbital.

2 total nodes - 1 angular node = 1 remaining node, so the 3p orbital has **one radial node**. This node is a sphere, centered at the nucleus.

Let’s sketch a picture of the nodes…
Here is what the two nodes look like. The nucleus is at the center of the sphere, and it lies on the plane.

If we cut through the atom on a plane perpendicular to the nodal plane, we slice through the two nodes in this fashion.

We can use this cross-section to sketch the shape of the actual orbital.
Here is a cross-section of the 3p orbital.

The gray areas show where the electron is most likely to be found; note that it cannot be on a node.
Now let’s construct our electron density plot.

We start the plot at the nucleus \((x = 0)\). There is a node at the nucleus, so the electron density is zero; our plot starts at the point \((0, 0)\).

As we move out from the nucleus (following the dotted arrow), the electron density rises, then drops to zero (as we cross the radial node), then rises again, and finally drops gradually toward zero as the distance becomes very large.

We expect a two-humped plot, similar to the one on the left.
Here is the actual electron density plot for the 3d orbital. Our drawing had the correct features (starting at the origin, two humps), but the left hump is much taller than the right hump.

This is the case for all electron density plots; if they have more than one peak, the peaks become shorter (and wider) as we move away from the nucleus.
Here is the electron density plot for a 3d orbital. The 3d orbital has two angular nodes (planes) that intersect at the nucleus, and it has no radial nodes. Therefore, the electron density plot starts at the origin (the electron density is zero at the nucleus, since the two nodes both pass through the nucleus), and it has a single hump (since there are no radial nodes).

A cross-section through a 3d orbital, showing the two nodes.
Here is a graph of the electron densities for all three \( n = 3 \) orbitals.

The orbitals all extend out to around 1000 pm, but otherwise they complement one another; where one orbital has a peak, at least one other has a node.

- The 3s orbital has its highest electron density at the nucleus, where both the 3p and 3d orbitals have nodes.
- The 3p orbital has its highest electron density at around 100 pm, where the 3s orbital has a node.
- The 3d orbital has its highest electron density around 300-400 pm, where both the 3s and 3p orbitals have nodes.
Electron density plots tell us where the electron is most likely to be found, but they give a distorted view of the typical distance between the electron and the nucleus.

For example, the 3p electron density plot seems to show us that the electron spends most of its time between 0 and 300 pm away from the nucleus. In fact, though, the electron spends only 11% of its time in this region.
Radial Probability

To get a clearer view of the typical distance between the electron and the nucleus, we must look at radial probability.

Radial probability is the probability that the electron will be found at a specific distance from the nucleus, but not at a specific location.

To make this clearer, let’s return to our picture of an atom…
We want to find the radial probability at 10 pm. Using our wave model, this is the overall fraction of the electron that is at a distance of 10 pm.

(Remember that in the wave model, the electron is “smeared out” over all of space.)
The tan ring shows all of the region that is 10 pm away from the nucleus. (The ring is 1 pm thick, so it is really the region from 9.5 pm to 10.5 pm away from the nucleus.)

However, this picture only shows two dimensions. Our “ring” should actually be a spherical shell.

Here is a view of a spherical shell that has been cut in half. The middle of the shell is empty.
Note the difference between electron density and radial probability. The electron density at 10 pm is the fraction of the electron that lies inside a 1 pm cube. The radial probability is the fraction of the electron that lies inside a spherical shell that is 1 pm thick. The shell is much larger than the cube, so the radial probability is much larger than the electron density.

**Electron density:** the fraction of the electron inside this cube.  

**Radial probability:** the fraction of the electron inside this spherical shell.
To calculate radial probability, we must multiply the electron density by the volume of the shell.

\[ \text{matter/pm}^3 \times \text{pm}^3 = \text{matter} \]

It can be shown that the volume of a shell that is 1 pm thick, located at a distance \( r \) from the center, is given by the following formula:

\[ V = 4\pi r^2 \]

Therefore, the radial probability at a distance \( r \) is:

**Radial probability** \( = 4\pi r^2 \psi^2 \)

*Strictly, the last two formulas are approximations, and they are accurate only when \( r \) is much greater than 1 pm.*
Here is a radial probability plot for the 1s orbital (the ground state) in a hydrogen atom.

What does this plot tell us?
The $x$ axis gives us the distance from the nucleus, just as it did for electron density plots.
The $y$ axis gives us the overall probability that the electron can be found at a specific distance from the nucleus.
For example, the probability that the electron is in a spherical shell that is 1 pm thick and 100 pm away from the nucleus is roughly 0.006 (0.6%).
This graph tells us that the electron is most likely to be about 50 pm away from the nucleus (actually 52.9 pm). The probability of the electron being in a 1 pm shell at this distance is slightly over 0.01 (1%).
There is a dramatic difference between the electron density plot and the radial probability plot for the 1s orbital.

The electron density is highest at the nucleus (r = 0 pm).

However, the radial probability drops to zero at this distance, and is highest at 52.9 pm.

How is this possible??
This picture may help you understand how a high electron density does not imply a high radial probability.

In the inner shell, the electron density is high (the dots are crowded together), but the radial probability is low (there are fewer dots).

In the outer shell, the electron density is low (the dots are widely spaced), but the radial probability is high (there are more dots).
If we shrink our distance to zero, the volume of the shell we are looking at drops to zero as well.

The radial probability is zero at the nucleus \((r = 0)\), because there is no space for the electron to be in!

At very small distances (but greater than zero), the radial probability is still very small, because the volume of our shell is very small. The small volume outweighs the high density.
The electron density and radial probability plots have one feature in common, though.

For both plots, the curve approaches zero as the distance from the nucleus becomes very large.
Here is the radial probability plot for the 2s orbital.

What does this graph tell us about the behavior of the electron in the 2s state?
The radial probability is very low close to the nucleus, becoming zero at the nucleus, exactly as we saw for the 1s orbital.

The reason is the same, too: as we approach the nucleus, the volume we’re looking at becomes very small, so there is little chance that the electron will enter that volume.
The radial probability also drops to zero at a distance of about 100 pm. This is where the 2s orbital has its radial node, the spherical surface where the electron is never found.
The curve has two peaks, with the highest one being at about 300 pm. This tells us that in the 2s orbital, the electron is most likely to be about 300 pm from the nucleus.
Comparing the electron density and radial probability plots for the 2s orbital, we see some differences and some similarities…
Both plots clearly show the radial node at about 100 pm.

Both plots also show that the electron is very unlikely to be a long way from the nucleus (the curves both approach zero at large distances).
The two plots differ as we get close to the nucleus. The electron density is high at the nucleus, but because the volume close to the nucleus becomes very small, the overall probability of finding the electron near the nucleus is also very small.
The graphs also differ in the relative sizes of the two “humps.” The electron density is highest at the inner hump (from 0 to 100 pm), while the radial probability is highest in the outer hump (beyond 100 pm).
Here are the electron density and radial probability plots for the 3s orbital.

We can see the same differences and similarities between these plots that we saw for the 2s plots.
Both plots show the two radial nodes (at about 100 pm and 400 pm), and both plots show that the probability of finding the electron at very large distances becomes vanishingly small.
The electron density is highest near the nucleus, but the radial probability is highest outside the second node. In the 3s orbital, the electron is most likely to be found roughly 700 pm away from the nucleus.

All electron density and radial probability plots behave similarly to these. The peaks in an electron density plot get smaller as we get farther from the nucleus, whereas the peaks in a radial probability plot get larger.
Here is the radial probability plot for the 2p orbital. It is a simple curve, with one large peak.

How does this plot compare with the electron density graph?
For the 2p orbital, the electron density and radial probability plots look rather similar.

This is quite different from what we saw with the 1s, 2s, and 3s orbitals, where the plots looked very different from one another…
Both plots start at the origin (0,0), but they do so for different reasons.

The electron density is zero at the nucleus because the 2p orbital has a node that passes through the nucleus.

The radial probability is zero at the nucleus both because of the node and because the volume at the nucleus is zero.

Note that the electron density climbs steeply as we move away from the nucleus (and away from the node), while the radial probability climbs much more gradually, because the volume remains low for a while.
Here are the electron density and radial probability plots for the 3p orbital.
As we saw for the 2p orbital plots, both of these graphs start at the origin. The 3p orbital, like all p orbitals, has an angular node (a plane) that passes through the nucleus, so its electron density is zero at the nucleus.

The radial probability is zero at the nucleus because of the node and because the volume drops to zero at the nucleus.
Both of these plots also show a radial node (a spherical surface) at about 350 pm.
The differences between these two plots are the same ones we saw for the 2s and 3s plots. In an electron density plot, the peaks get smaller as we move away from the nucleus, while in a radial probability plot, the peaks get larger.
How do radial probability plots for different orbitals compare to one another?

Let’s start by looking at what happens when we compare orbitals with different values of n.

We’ll begin by comparing the 1s, 2s, and 3s orbitals.
When we compare these three orbitals, we clearly see the differences in size. The 1s orbital is the smallest, extending out to about 300 pm. The 2s orbital extends out to around 800 pm. The 3s orbital is the largest, extending out to around 1400 pm.

Once again, we see that as $n$ increases, the effective size of the orbital increases.
We can also see that the number of nodes increases as n increases.

The 1s orbital has no nodes.

The 2s orbital has one node, at about 100 pm.

The 3s orbital has two nodes, at around 100 pm and 400 pm.
The 1s orbital has a single narrow, high peak. In this orbital, the electron spends most of its time very close to the nucleus.

In contrast, the 3s orbital has three broad, low peaks. In this orbital, the electron spreads itself out over a much larger region, so it does not spend much time at any one distance.

The 2s orbital is intermediate, as you might expect.
As we saw earlier, in any radial probability plot that has more than one peak, the peaks get higher as the distance from the nucleus increases. This means that the electron is most likely to be found beyond the outermost radial node.
Here is a comparison of the radial probability plots for the 2p and 3p orbitals. Again, we see that…

• The orbitals get larger as n gets larger.

• The smaller orbital has a taller, narrower peak, because its electron spends most of its time close to the nucleus.
Here is a comparison of the radial probability plots for the 3s, 3p, and 3d orbitals.

What can we tell from this graph?
First, we notice that all three orbitals extend out about the same distance (around 1500 pm). **Orbitals with the same value of n are about the same size.**
Second, we notice that all three curves start at the origin. All radial probability plots start at the origin, because the volume is zero at the nucleus.

The 3p and 3d orbitals have angular nodes that pass through the nucleus, so the electron can not be found at the nucleus in any case.

For the 3s orbital, though, the electron density is highest at the nucleus.

Radial probability plots cannot tell us whether there is a node at the nucleus.
Third, we notice that the orbitals have different numbers of peaks, because they have different types of nodes.

All $n = 3$ orbitals have two nodes.

The 3s orbital has two radial nodes (spheres), so we see two points where the radial probability is zero (100 pm and 400 pm).

The 3p orbital has one angular node (a plane, which doesn’t show up on a radial probability plot), and one radial node at 300 pm.

The 3d orbital has two angular nodes (planes), which don’t show up on a radial probability plot.
We also notice that each orbital has its peaks in different places from the others. The 3d orbital has its highest radial probability closer to the nucleus than the other two orbitals. However, in each orbital, the **average distance between the electron and the nucleus is the same** (as required by the fact that the three orbitals have equal energies). The inner peaks of the 3p and 3s orbitals exactly balance the closer distance of the main peak for the 3d orbital.
Let’s see if we can identify an orbital!

The graph below is a plot for an orbital that has \( n = 4 \).

What type of graph is this (electron density or radial probability), and what orbital does the graph represent?

See if you can figure it out before you go on…
We notice that the peaks get bigger as the distance from the nucleus increases.

This fact tells us that we are looking at a **radial probability** graph.
We were told that this orbital has $n = 4$, so we know that the orbital has three nodes.

If we can figure out the number of angular nodes (nodal planes), we can identify the orbital.

Remember!!

0 angular nodes means the orbital is an s orbital ($\ell = 0$).

1 angular node means the orbital is a p orbital ($\ell = 1$).

2 angular nodes means the orbital is a d orbital ($\ell = 2$).

3 angular nodes means the orbital is an f orbital ($\ell = 3$).
We cannot determine the number of angular nodes from a radial probability plot, because angular nodes pass through the nucleus, where the radial probability is always zero anyway.

However, we can determine the number of radial nodes from the shape of the graph.

Once we know the number of radial nodes, simple subtraction will give us the number of angular nodes!
There are three points (other than the origin) where the graph touches the $x$ axis.

Therefore, this orbital must have three radial nodes.
Since the orbital has three radial nodes, it must have **no angular nodes**!

3 total nodes – 3 radial nodes = 0 angular nodes
So we now know that our orbital is an **s orbital** (since it has no angular nodes) and that it has **n = 4**.

Putting these together, we conclude that this is the radial probability plot for a **4s orbital**!

...and we are correct!!!
Here is one more for you to try. Can you identify the type of graph and the specific orbital here?
The graph type is easy. Since the peaks get smaller as the distance increases, this is an *electron density plot*.
We can see two radial nodes on this plot, as shown by the arrows.
Since this is an electron density plot, we can also tell whether there are any angular nodes.

Since the graph starts at the origin, there must be at least one angular node. Only angular nodes pass through the nucleus.
Here’s the catch, though. We can tell that there is at least one angular node, but we cannot tell how many angular nodes there are. Our orbital might have…

…two radial nodes and one angular node.

…two radial nodes and two angular nodes.

…two radial nodes and three angular nodes.

etc…
So in this case, we cannot determine which orbital the graph represents.

We can, however, list some possibilities.
The table below lists the first four possible orbitals that this graph could represent.

<table>
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<th>Number of radial nodes</th>
<th>Number of angular nodes (equals ( \ell ))</th>
<th>Total number of nodes</th>
<th>Value of n (equals 1 + total nodes)</th>
<th>Orbital type</th>
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