Linear fit via least-squares (summary)

To fit a straight line \( y = mx + b \) to \( N \) data points \((x_1, y_1), (x_2, y_2), \ldots, (x_N, y_N)\):

\[
m = \frac{N \left( \sum_i x_i y_i \right) - \left( \sum_i x_i \right) \left( \sum_i y_i \right)}{\Delta}
\]
\[
b = \frac{\left( \sum_i y_i \right) \left( \sum_i x_i^2 \right) - \left( \sum_i x_i y_i \right) \left( \sum_i x_i \right)}{\Delta}
\]

where

\[
\Delta = N \left( \sum_i x_i^2 \right) - \left( \sum_i x_i \right)^2
\]

To calculate uncertainties in the fit,

\[
\delta m = \sqrt{\frac{\sigma^2 N}{\Delta}}
\]
\[
\delta b = \sqrt{\frac{\sigma^2 \left( \sum_i x_i^2 \right)}{\Delta}}
\]

where

\[
\sigma^2 = \frac{1}{N - 2} \left( \sum_i (mx_i + b - y_i)^2 \right)
\]

\( \sigma^2 \) can also be calculated via

\[
\sigma^2 = \frac{1}{N - 2} \left( \sum_i y_i^2 - m \left( \sum_i x_i y_i \right) - b \left( \sum_i y_i \right) \right)
\]

as long as exact values are used for all quantities — the round-off errors in this formula are huge.